Convergence and Energy Analysis for Iterative Adaptive ON-OFF Control of Piezoelectric Microactuators

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***Abstract*— A technique is presented for estimating the number of iterations needed for convergence of iterative adaptive ON-OFF controllers to optimal switching times, for certain controllers proposed for managing stepping motion in autonomous microrobots. An upper bound on output error as a function of error in ON-OFF switching times is obtained, and lower bounds on the change in switching times from iteration-to-iteration are used to estimate output error evolution. The simulation and experimental results from the test case of a piezoelectric microrobotic leg joint indicate reasonable agreement between estimated and actual error. The use of convergence estimates to improve controller design with respect to total energy consumption is then described.**

***Index Terms*— Iterative control, microsystems, switching control.**

I. INTRODUCTION

ACTUATORS in many microelectromechanical system applications act as small capacitive loads and are desired to operate at very low power levels. When such microactuators are incorporated into feedback control systems under conventional approaches, driving and sensing circuitry may easily consume more energy than actuators. For example, analog driving voltages are comparatively inefficient when driving piezoelectric actuators [1]. Analog inputs are therefore preferably replaced by pulse-width modulation (PWM) or, under even power constraints, low frequency ON-OFF switching inputs [2]. Similarly, sensing circuitry sampling at high rates to capture the full dynamics of high-bandwidth microactuators is also expensive from a power consumption standpoint.

One low-power approach to controlling microactuator motion is to perform iterative adaptive ON-OFF control with limited numbers of sensor measurements when those motions can be expected to repeat many times, as for autonomous walking microrobotics. The authors have previously described two iterative adaptive ON-OFF controllers for thin-film piezoelectric microrobotic leg joints, one based on stochastic gradient approximation [3] and the other on deterministic heuristic switching rules [4], [5]. In both cases, eventual convergence of switching times to at least locally optimal values was guaranteed for some limited range of initial switching time selections and adaptation gains. No explicit model of the system is required for adaptation, though a nominal model is useful to ensure that adaptation gains are stable. The rate at which the switching time selections converge to optimal values could not be, however, predicted. Thus, the controllers can limit the energy expended during each iteration (to $\sim$10%–30% of an analog controller with comparable response) but it does not make any predictions of total energy required to identify the optimal switching times. The aim of this paper, then, is to estimate the rate of convergence of these ON-OFF iterative adaptive controllers to optimal switching instances, then to show how this knowledge can be used to improve overall power or energy consumption by choice of sampling rates and/or number of switched inputs permitted.

The majority of previous researches on iterative adaptive control have dealt with dynamic systems having analog inputs and a single-system model, although iterative control of switched systems is becoming more common [6], [7]. Convergence times in these applications were measured only empirically, but have typically been found to be fast (on the order of tens of iterations to reach very small error margins). Other iterative adaptive controllers with switched dynamics and reportedly fast convergence rates have been developed based on neural networks, multiple models, or both [8]–[10]. In all of these researches, analog inputs to the system are still permitted, though either the system or controller dynamics are switched systems. A lesser number of iterative controllers permit strictly ON-OFF or high-low (bang–bang) inputs to uncertain dynamic systems [11], [12]. In both cases, predictions of convergence speed could not be made, though some suggestions for improvement were provided. It is also worth noting that iterative approaches are often used for off-line optimization of switching inputs to systems, either ON-OFF time optimal control [13], [14] or more general hybrid systems [15], [16], though of course these are strategies for control of systems with known dynamics.

Explicit methods for predicting or guaranteeing convergence rates of iterative adaptive controllers, meanwhile, are known for a variety of well-known classes of iterative learning or adaptive controllers [17]–[21], [26]. These do not, unfortunately, include switching control systems at this time. For instance, Hillenbrand and Pandit [19] could guarantee an exponential convergence rate for an iterative learning controller for an linear time-invariant (LTI) discrete-time system. This also allowed relatively slow sampling rates to be used, a feature of the authors' ON-OFF controllers to be analyzed in this paper, but inputs were not constrained. Other analyzes of convergence rates have introduced inequality constraints and/or nonlinearities on certain signals [18], [26].

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Thus, it remains difficult to make predictions of convergence rates of iterative control for switched systems. This paper describes a limited method for estimating the number of iterations needed to reach specified error levels for certain simple switching controllers, namely the ON-OFF controllers for microrobotic leg joints described in previous researches. The approach makes use of the binary nature of ON-OFF inputs (0 or 1) and knowledge about adaptation behavior in the known regions of convergence. Analysis will be applied to two ON-OFF adaptive controllers (the heuristic adaptive approach and stochastic gradient approximation approach). The resulting method for predicting output error as a function of iteration number is found to have reasonable agreement with simulated and experimental test cases.

II. System Model

The dynamic system to which ON-OFF adaptive controllers will be applied is assumed to be a single-input, single-output (SISO), continuous time, and LTI system with known nominal and bounded (but unknown) perturbed dynamics. Convergence analysis will be performed using nominal system dynamics, described by state vectors, an input, an output, and a measured variable $\mathbf{x} \in \mathbf{R}^M$, $u \in \mathbf{R}$, $z \in \mathbf{R}$, and $y_n \in \mathbf{y} \in \mathbf{R}^n$, respectively, as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \\ z(t) &= \mathbf{C}\mathbf{x}(t) \\ y_n &= z(t_n), \quad n = 0, \ldots, N \end{aligned} \tag{1}$$

where $\mathbf{A}$ is a state matrix, $\mathbf{B}$ is an input matrix, and $\mathbf{C}$ is a linear relationship between states and an output. Note that the measured output, y, is obtained only at a fixed number of N sampling instants occurring at sampling instances t_n between initial time $t_0 = 0$ and final time t_N. The state, output, measurement, and input vectors of time duration t_f at the kth iteration are denoted by $\mathbf{x}^k, z^k, \mathbf{y}^k$, and u^k, respectively. It should be noted that this system model is used only for convergence analysis, typically to help select adaptation gains, while the proposed techniques do not use any model inside the controller for adaptation.

The system in (1) is taken to be asymptotically stable with equilibrium points $r_{eq} \in \mathbf{R}$ and 0 under on (i.e., $u = 1$) and off (i.e., $u = 0$) conditions satisfying

$$\begin{cases} \mathbf{A}\mathbf{x}(t) + \mathbf{B} = 0, & \text{iff } \mathbf{x} = \left[r_{\text{eq}}\ 0 \cdots 0 \right]^T \\ \mathbf{A}\mathbf{x}(t) = 0, & \text{iff } \mathbf{x} = \left[0\ 0 \cdots 0 \right]^T. \end{cases} \tag{2}$$

In addition, it is assumed that at $t^k = t_0 = 0$

$$\mathbf{x}^k = \mathbf{0} \tag{3}$$

for all iterations k due to stability and satisfactory time between motions for the system to return to its off-equilibrium state.

III. General Controller Structure

In an ON-OFF controller, a system has only two possible values for its input, $u(t)$: on ($u = 1$) and off ($u = 0$). Then, using a vector $\boldsymbol{\tau}$ for switching instances and sampling instant, t_n, the input $u(t)$ can be described over time by

$$u(t) = \begin{cases} 1, t_n < t < \tau_{1,n+1}, n = 0, \ldots, N-1 \\ 0, \tau_{2q-1,n} < t < \tau_{2q,n}, q = 1, \ldots, Q/2 \text{ and } n = 1, \ldots, N \\ 1, \tau_{2q,n} < t < \tau_{2q+1,n}, q = 1, \ldots, Q/2 \text{ and } n = 1, \ldots, N \end{cases} \tag{4}$$

where $\boldsymbol{\tau}$ is defined as

$$\begin{aligned} \boldsymbol{\tau}^T &= \left[\tau_{1,1} \cdots \tau_{Q,1}\ \tau_{1,2} \cdots \tau_{Q,2} \cdots \tau_{1,N} \cdots \tau_{Q,N} \right] \\ 0 &\le \tau_{1,1} \\ \tau_{q-1,n} &\le \tau_{q,n}, n = 1, \ldots, N \text{ and } q = 2, \ldots, Q \\ \tau_{q,n-1} &\le \tau_{q,n}, n = 2, \ldots, N \text{ and } q = 1, \ldots, Q \\ \tau_{Q,n} &\le \tau_{1,n+1}, n = 1, \ldots, N-1 \\ \tau_{q,n} &< t_n, n = 1, \ldots, N \text{ and } q = 1, \ldots, Q \end{aligned} \tag{5}$$

where t_n is a sampling instant which is considered to be chosen at a regular sampling time T_s, i.e., $t_n = n \cdot T$. Note that the switching times are grouped by their sampling instant, t_n, with Q switching times in each of N sampling instances. It is noted that the sample time T_s may optionally be slower than Nyquist, since only error at a finite number of time points is regulated and no system model is explicitly identified within these controllers, but intersample behavior can be very poor and feasible states may be limited. In this case, all behavior of a system may not be observed, but a limited number of measurements may be available and outputs at those instance potentially regulated.

The controllers' goal is to minimize a positive definite objective function, J, that is a function of measurement errors

$$J = \begin{cases} f(\boldsymbol{\varepsilon}) = 0, & \boldsymbol{\varepsilon} = 0 \\ f(\boldsymbol{\varepsilon}) > 0, & \boldsymbol{\varepsilon} \neq 0 \end{cases} \tag{6}$$

where $\boldsymbol{\varepsilon}$ is the error vector of output measurements, such that $\boldsymbol{\varepsilon}^k = [\varepsilon_0^k\ \varepsilon_1^k\ \ldots\ \varepsilon_N^k]^T = [y_0^k - r_0 y_1^k - r_1 \ldots y_N^k - r_N]^T$ in iteration k and $\mathbf{r}$ is reference vector such that $\mathbf{r} = [r_0\ r_1 \ldots r_N]$.

Adaptation of the control input is performed by adapting the switching inputs in $\boldsymbol{\tau}$ as a function of error measurements

$$\boldsymbol{\tau}^{k+1} = \boldsymbol{\tau}^k + \boldsymbol{\Psi}^k \left(\varepsilon_1^k, \varepsilon_2^k, \ldots, \varepsilon_N^k \right) \tag{7}$$

where in the controllers developed $\boldsymbol{\Psi}$ takes the form of linear gains $\psi_{q,n}$ from measurement n to switching time (q, n). The gains may vary with iteration or on a random perturbation, and are also bounded in size, as below

$$\begin{aligned} \left(\boldsymbol{\Psi}^k\right)^T &= \left[\Psi_{1,1}^k \cdots \Psi_{Q,1}^k\ \Psi_{1,2}^k \cdots \Psi_{Q,2}^k \cdots \Psi_{1,N}^k \cdots \Psi_{Q,N}^k \right] \\ \left(\Psi_{q,n}^k\right)_{\min} &\le \Psi_{q,n}^k \le \left(\Psi_{q,n}^k\right)_{\max}. \end{aligned} \tag{8}$$

IV. Estimation of Convergence Time

The objective of the convergence analysis to be performed is to estimate the error in output measurements, $\boldsymbol{\varepsilon}^{\mathrm{k}}$, in iteration k given a maximum initial error in iteration 0. Strictly speaking, in the convergence rate estimation procedure to follow, a maximum bound on output error is found for adaptation of switching times to their optimal value for the nominal system. This convergence rate is taken as a representative estimate for convergence rate to optimal values for perturbed versions of systems.

A. Requirements for Convergence Rate Estimation

It is assumed that the adaptive controller of the form described in (4) and (7) has the following properties.

Property 1: If a minimizing solution, $\boldsymbol{\tau}^*$, of the input parameter vector for the object function (6) exists, the adaptive algorithm begins in a region of convergence to that solution such that

$$\lim_{k\to\infty} \left\| \boldsymbol{\tau}^k - \boldsymbol{\tau}^* \right\| = 0. \tag{9}$$

Property 2: The adaptation rule in (9) can be designed to satisfy

$$E\left[\left|\tau_{i,j}^{k+1} - \tau_{i,j}^*\right|\right] < E\left[\left|\tau_{i,j}^{k} - \tau_{i,j}^*\right|\right] \qquad \forall\, i, j. \tag{10}$$

These properties imply an important behavior of an adaptive controller for ensuring convergence: that there exists a minimizing $\boldsymbol{\tau}^*$ with a domain of attraction in terms of $\boldsymbol{\tau}$. If $\tau_{n,q}^1$ for all n and q is then chosen to be within the domain of attraction, then a controller can be designed such that all input parameters in $\boldsymbol{\tau}$ are steadily converging to their optimal value $\boldsymbol{\tau}^*$.

Equation (10), it must be noted, is the restrictive assumption on the current convergence analysis, and can be difficult to verify for problems with large numbers of both measurements and switching instances. However, two controllers where these conditions can be satisfied (either deterministically or in their expected value) will be presented in Sections V-A and V-B.

To derive the convergence rate, the L^1- and $L^\infty-$norm, denoted as $\|f(t)\|_{1,t\in[t_n,t_{n+1}]}$ and $\|f(t)\|_{\infty,t\in[t_n,t_{n+1}]}$, respectively, are used for a piecewise continuous function $f(t)$ on interval $t \in [t_n, t_{n+1}]$. These norms hold with respect to Holder's inequality [22].

B. Evolution of Output Error

In this section, a bound on output error of the nominal system as switching times adapt to their optimal values is derived as a function of adaptation rules and system dynamics.

Theorem 1: Given the nominal system (1) and switching time adaptation in (4) and (7), if the adaptation rule satisfies Properties 1 and 2 in (9) and (10), then the evolution of output error in each iteration can be bounded by the recursive formula

$$E\left[\left|\boldsymbol{\varepsilon}^k\right|\right] \le \mathbf{w}^k\Big|_{\max} \tag{11}$$

$$\mathbf{w}^{k+1}\Big|_{\max} \le \left\{I - \mathbf{F}E[\boldsymbol{\Gamma}] + (\mathbf{F}E[\boldsymbol{\Gamma}])^2\right\}\mathbf{w}^k\Big|_{\max} \tag{12}$$

where $\mathbf{w}^k|_{\max}$ is an upper bound to a set of vector/matrix inequalities and $\mathbf{F}$ and $\boldsymbol{\Gamma}$ are matrices computed from the nominal system dynamics and adaptation rule, respectively, as derived below.

To derive Theorem 1, first the adaptation algorithm in (7) is converted to a time-dependent input formulation. Let $u_n^k(t)$ be a piecewise continuous input function in the nth sampling period of the kth iteration satisfying (4) and (5), defined on a time interval $t \in [t_{n-1}, t_n)$. Then, let $u_n^*(t)$ be the piecewise optimal input function corresponding to the vector $\boldsymbol{\tau}^*$, where $\boldsymbol{\tau}^*$ is the optimal switching time vector satisfying $\boldsymbol{\tau}^* = \arg\min_\tau J(\boldsymbol{\tau})$

Input error, $\eta_n(t)$, on sampling interval $t \in [t_{n-1}, t_n)$ can be defined as

$$\eta_n^k(t) \equiv u_n^k(t) - u_n^*(t) = \sum_{n=1}^{N} u_n^k(t) - \sum_{n=1}^{N} u_n^*(t) \tag{13}$$

using the actual and desired time domain inputs, $u^k(t)$ and $u^*(t)$, over the full time interval $t \in [t_0, t_N)$ in iteration k.

On an interval $t \in [t_{n-1}, t_n)$ over the kth and $(k+1)$th iterations, the norm of input error evolves according to

$$\begin{aligned}\left\|\boldsymbol{\eta}_n^{k+1}\right\|_{1,[t_{n-1},t_n]} = -t_{n-1} &+ \sum_{q=1}^{Q/2}\left(\boldsymbol{\tau}_{2q-1,n}^k + \Psi_{2q-1,n}^k - \boldsymbol{\tau}_{2q,n}^k - \Psi_{2q,n}^k\right)\\ &- \sum_{q=1}^{Q/2}\left(\boldsymbol{\tau}_{2q-1,n}^* - \boldsymbol{\tau}_{2q,n}^*\right)\end{aligned} \tag{14}$$

with the change in the input error norm from k to $k+1$ arises entirely from the switching time updates $\boldsymbol{\Psi}^{\mathrm{k}}$ and (11) ensuring that $\mathrm{sign}[(\tau_{2q-1,n}^k - \tau_{2q-1,n}^*)^*\Psi_{\mathrm{2q-1,n}}^{\mathrm{k}}] < 0$ and sign $[(\tau_{2q,n}^k - \tau_{2q,n}^*)^*\Psi_{\mathrm{2q,n}}^{\mathrm{k}}] > 0$ in the individual terms. For instance, an off-switch occurs earlier, $\Psi_{\mathrm{2q-1,n}}^{\mathrm{k}} < 1$, if it is currently switching too late, $\boldsymbol{\tau}_{\mathrm{2q-1,n}}^{\mathrm{k}} - \boldsymbol{\tau}_{\mathrm{2q-1,n}}^* > 0$. Therefore, (14) can be bounded as

$$\left\|\boldsymbol{\eta}_{\mathrm{n}}^{\mathrm{k}+1}\right\|_{1,[t_{n-1},t_n]} \le \left\|\boldsymbol{\eta}_{\mathrm{n}}^{\mathrm{k}}\right\|_{1,[t_{n-1},t_n]} - \sum_{q=1}^{Q}\sum_{i=1}^{n}\left|\frac{\partial\Psi_{\mathrm{q,n}}^{\mathrm{k}}}{\partial\varepsilon_i}\right|_{\min}\left|\varepsilon_i^k\right| \tag{15}$$

where $|\partial\Psi_{q,n}/\partial\varepsilon_i|_{\min}$ is the magnitude of the minimum possible adaptation gain defined in (8) from measurement error i to switching time q in period n. Then, evolution of the norm of input error can be formulated in matrix representation as (in deterministic or nondeterministic cases) as

$$\begin{gathered}\overline{\boldsymbol{\eta}}_{L1}^{\mathrm{k}+1} - \overline{\boldsymbol{\eta}}_{L1}^{\mathrm{k}} \le -\boldsymbol{\Gamma}\left|\boldsymbol{\varepsilon}^{\mathrm{k}}\right|\\ E\left[\overline{\boldsymbol{\eta}}_{L1}^{\mathrm{k}+1}\right] - E\left[\overline{\boldsymbol{\eta}}_{L1}^{\mathrm{k}}\right] \le -E[\boldsymbol{\Gamma}]\,E\left[\left|\boldsymbol{\varepsilon}^{\mathrm{k}}\right|\right]\end{gathered} \tag{16}$$

where entries $\boldsymbol{\Gamma}_{i,j}$ in $\boldsymbol{\Gamma} \in \mathbf{R}^{N\times N}$ are computed from (14) and

$$\overline{\boldsymbol{\eta}}_{L1}^k \equiv \left[\ \left\|\boldsymbol{\eta}_1^k\right\|_{1,[t_0,t_1]}\ \left\|\boldsymbol{\eta}_2^k\right\|_{1,[t_1,t_2]}\ \cdots\ \left\|\boldsymbol{\eta}_N^k\right\|_{1,[t_{N-1},t_N]}\ \right]^T.$$

For the next phase of the derivation, the relationship of the norm of input error, $\|\eta_n^k\|_{1,[t_{n-1},t_n]}$, to output error in each sampling interval is investigated. Output error can be bounded using (1) and norm of input error, $\|\eta_1^k\|_{1,[t_0,t_1]}$. Applying the L^1 norm to the output error, ε_n^k, and then Holder's inequality, a bound for the nth sampled output error over kth iteration is obtained as

$$\left\|\boldsymbol{\varepsilon}_{\mathrm{n}}{}^{k}\right\|_{1,[t_0,t_n]} \le t_n \sum_{i=1}^{n} \left\|\left(\mathbf{C}e^{\mathbf{A}(t_n-t)}\mathbf{B}\right)\right\|_{\infty,t\in[t_{i-1},t_i]} \left\|\eta_i^k\right\|_{1,[t_{i-1},t_i]}. \tag{17}$$

To estimate convergence rate of the norm of output error, a matrix representation for (17) may be formulated as

$$\left|\boldsymbol{\varepsilon}^k\right| \le \mathbf{F}\overline{\boldsymbol{\eta}}_{L1}^k \equiv \mathbf{w}^k \text{ or } E\left[\left|\boldsymbol{\varepsilon}^k\right|\right] \le \mathbf{w}^k \tag{18}$$

where entries $F_{i,j}$ in $\mathbf{F} \in \mathbf{R}^{N\times N}$ are taken from (17) and w^k is defined to be a bound on the true error vector, $\boldsymbol{\varepsilon}^k$, in iteration k.

Returning to the nominal system, the evolution of the output error can be bounded in an individual iteration step by combining (16) and (18)

$$\begin{aligned}\mathbf{w}^{k+1} &= \mathbf{F}\overline{\eta}_{\mathbf{L1}}^{k+1} = \mathbf{F}\overline{\eta}_{L1}^{k} + \mathbf{F}\left(\overline{\eta}_{L1}^{k+1} - \overline{\eta}_{L1}^{k}\right)\\ &\le \mathbf{w}^k - \mathbf{F}\Gamma\left|\boldsymbol{\varepsilon}^k\right|.\end{aligned} \tag{19}$$

However, because only a bound on $|\boldsymbol{\varepsilon}^k|$ is known in iteration k, to numerically calculate error estimates, a substitution of $\mathbf{w}^k$ for $|\boldsymbol{\varepsilon}^k|$ is finally done to complete the error estimator and predict convergence. Since $\mathbf{w}^k \ge |\boldsymbol{\varepsilon}^k|$, direct substitution of $\mathbf{w}^k$ into (18) can lead to an underestimate of $\mathbf{w}^{k+1}$ and $|\boldsymbol{\varepsilon}^{k+1}|$. For a conservative estimate, assume a scenario where $\mathbf{w}^k - \mathbf{F\Gamma w}^k$ is an underestimate

$$\left|\boldsymbol{\varepsilon}^{k+1}\right| > \mathbf{w}^k - \mathbf{F\Gamma w}^k. \tag{20}$$

If true, then given uniform convergence from Property 2 in (10), $|\boldsymbol{\varepsilon}^k| \ge \mathbf{w}^k - \mathbf{F\Gamma w}^k$ is also true, which with (18) produces

$$\mathbf{w}^{k+1} \le \mathbf{w}^k - \mathbf{F\Gamma}\left(\mathbf{w}^k - \mathbf{F\Gamma w}^k\right). \tag{21}$$

If (20) is not true, as for example $|\boldsymbol{\varepsilon}^{k+1}| < \mathbf{w}^k - \mathbf{F\Gamma w}^k$, (21) remains an upper bound on $\mathbf{w}^{k+1}$, and thus (21) is used to define a dynamic convergence bound on adaptation to optimal switching times, $\mathbf{w}^k|_{\max}$, $\mathbf{w}^{k+1}|_{\max} = \{I - \mathbf{F\Gamma} + (\mathbf{F\Gamma})^2\}\mathbf{w}^k|_{\max}$, or $\mathbf{w}^{k+1}|_{\max} = \{I - \mathbf{FE}[\Gamma] + (\mathbf{FE}[\boldsymbol{\Gamma}])^2\}$ thus completing the derivation of Theorem 1. The same conclusion may be reached by treating (18) and (19) as defining a linear programming problem.

The implication of Theorem I is that the error (deterministic case) or expected error (stochastic case) in switching time has a monotonically decreasing upper bound, though the error itself is not guaranteed to decrease monotonically. In practice, this bound can be tight when motion durations are short and dynamics are comparatively low order, as shown in the case studies in Section V. This can allow the number of iterations required to reach a desired level of error to be predicted with reasonable accuracy. This may aid in controller design, either through selection of sampling rates or a target number of switching instances (to be illustrated in Section V), or by guiding design Ψ to obtain a specific $\mathbf{F\Gamma}$, provided Property 2 is maintained.

The desirable features above depend on being able to find $\mathbf{F}$ and $\boldsymbol{\Gamma}$, which can be a significant limitation as system order and/or complexity increases. This also means that a nominal system model must be available. While the adaptive controllers for which this estimation is intended also require a nominal model to set adaptation parameters, one of their benefits is that they can accommodate model error, including bounded nonlinearities while requiring no explicit model data in the controller itself. Conceptually, model error can be incorporated into the bound on convergence in (21) by expanding $\mathbf{F}$ to allow for worst case state error given bounded nonlinearities in state equations, similar to the methods used to prove convergence of the adaptive controller for a underdamped second-order system in [4]. This would dramatically increase the complexity of computing $\mathbf{F}$, which is already cumbersome. While simplified versions of $\mathbf{F}$ and $\boldsymbol{\Gamma}$ were tested by the authors, in the test cases examined, further simplification from the forms provided in (16) and (18) led to conversion rate estimates that were conservative by 20 times or more.

TABLE I
COEFFICIENTS OF POWER MODEL

Coefficient	E_{switch} (μJ)	p_1 (V)	p_2 (μW/Hz)	p_3 (μW)
Specific Values	0.15	15	1.6	106

TABLE II
TARGET REQUIREMENTS OF THE CASE STUDIES

Items	Max. angle of legs (rad)	Control accuracy (rad)	Power budget (mW)	Energy budget (mWh)
Specific Values	0.42	0.04	0.4 ~ 4	0.8 ~ 8

V. CASE STUDIES

The eventual goal of the convergence estimator for an ON-OFF iterative controller is to predict the number of iterations required to reach a desired level of output error. To estimate energy dissipation in these case studies, power models for a capacitive sensing circuit [29] and switching driving circuit are used as $P_{\text{sen}} = p_1 I_q + p_2/T_s + p_3$ and $P_{\text{dri}} = E_{\text{switch}} f_d$, respectively. Here, p_1, p_2, and p_3 are constants of power consumption of components in circuitry, I_q is quiescent current of an operational amplifier (op-amp), T_s is sample time, E_{switch} is the energy needed to charge the switching circuit. Values for parameters regulating power consumption are shown in Table I. The specific sensing circuit used to compute numerical power estimates is a differential capacitive sensing circuit with frequency modulation using a low-power instrumentation op-amp [27] and a multiplier [28].

The motivating application for these case studies is the regulation of microrobotic leg joints consisting of several thin-film lead-zirconate-titanate (PZT) rotational actuators in a series array, delivering up to 10° rotation from a 3–mm-long leg. As prototype actuators do not have embedded sensing, position measurements were taken with a high-speed camera through an optical microscope. Target performance for future microrobot walking is shown in Table II. Images of leg joints and a description of the experimental test setup can be found in [3] and [4].

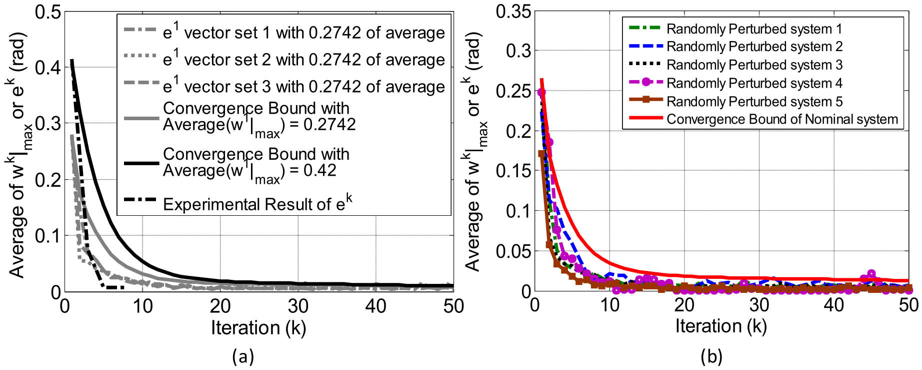

Fig. 1. (a) Convergence rate of several mean initial errors of output for the adaptive controller with heuristic search (red lines), and experimental versus estimated convergence (blue lines). (b) Several perturbed systems of actual convergence rate for the adaptive controller with heuristic search.

A. Case I: Heuristic Adaptive ON-OFF *Controller*

1) Test System Description: For the first test case, the controller to be analyzed is designed to adapt the timing of three parameters describing an ON-OFF input signal to produce a step-and-hold type response from the robot leg dynamics, as was previously described in [4]. In this controller, the number of sampling instances, N, can be varied, but switches are fixed to Q = 1 per sample. To analyze the controller with respect to convergence rate, the adaptation rules can be rewritten as

$$\begin{aligned}\Psi_{1,1}^{k+1} &= -\gamma_p \varepsilon_1^k \\ \Psi_{2,1}^{k+1} &= \Psi_{2,n}^{k+1} = \gamma_s \varepsilon_1^k - \gamma_s \varepsilon_2^k, \quad n = 2, \ldots, N \\ \Psi_{1,n}^{k+1} &= \gamma_s \varepsilon_1^k - \gamma_s \varepsilon_2^k - \frac{\gamma_d \varepsilon_d^k}{f_d(N-1)}, \quad n = 2, \ldots, N \qquad (22)\end{aligned}$$

where, ε_d^k is an average of ε_3^k to ε_N^k. Thanks to *prior* analysis of convergence regions, Property 1 from (9) is known to be satisfied. Property 2 from (10) is only fully guaranteed for measurement period 1, since switch times in periods 2 to N are only adjusted on average, although the convergence estimator, as will be seen, still provides a good estimate of error in each iteration. Specific limitations and important notes for using this approach are described in detail in [4].

2) Convergence Rate Analysis: The nominal system dynamics for the robot leg to be studied were taken from a prototype thin-film PZT actuator [4], approximated as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-14 & -4\times 10^4\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}4\times 10^4\\ 0\end{bmatrix}u(t), z(t) = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}. \qquad (23)$$

Convergence rates were first compared with analytical predictions in simulation. To analyze the above ON-OFF controller, by applying the adaptation gains from (22) to create matrix $\mathbf{\Gamma}$ in (15), and evaluating matrix $\mathbf{F}$ from (16) using the system dynamics in (26), an approximate error vector w^k may be quickly generated. This could in turn be related to the objective function J. The matrix $\mathbf{\Gamma}, \mathbf{F}$, and their value for the simulated example are formulated as shown in (24) at the bottom of the page.

Gray lines in Fig. 1(a) show the speed with which the adaptive rule (22) produced convergence to near zero error while the estimated error maintains similar values. In this example, values used for γ_p, γ_s, γ_d, and f_d are 0.008, 0.005, 0.3, and 100 Hz, respectively. Gray lines in Fig. 1(a) also show the actual convergence rate for several initial set of output errors but same average valued initial error. The convergence estimator with nominal dynamics is also compared with actual convergence rate with several perturbed dynamics in Fig. 1(b). As can be seen, the convergence estimator provides a close upper bound to the actual convergence rate. At very low error levels, lack of variable adaptation gains leads to oscillation of error in the simulated system, compared with the convergence estimator.

$$\mathbf{\Gamma} = \left\|\begin{bmatrix} -\gamma_p & 0 & 0 & \cdots & 0 \\ \gamma_s & -\gamma_s - \frac{\gamma_d}{f_d(N-1)} & -\frac{\gamma_d}{f_d(N-1)} & \cdots & -\frac{\gamma_d}{f_d(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ \gamma_s & -\gamma_s - \frac{\gamma_d}{f_d(N-1)} & -\frac{\gamma_d}{f_d(N-1)} & \cdots & -\frac{\gamma_d}{f_d(N-1)} \end{bmatrix}\right\| = \begin{bmatrix} 0.005 & 0 & 0 & \cdots & 0 \\ 0.008 & 0.0088 & 0.008 & \cdots & 0.008 \\ \vdots & \vdots & \vdots & & \vdots \\ 0.008 & 0.0088 & 0.008 & \cdots & 0.008 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} t_1 \left\|\mathbf{C}e^{\mathbf{A}(t_1-t)}\mathbf{B}\right\|_{\infty,t\in[t_0,t_1]} & 0 & \cdots & 0 \\ t_2 \left\|\mathbf{C}e^{\mathbf{A}(t_2-t)}\mathbf{B}\right\|_{\infty,t\in[t_0,t_1]} & t_2 \left\|\mathbf{C}e^{\mathbf{A}(t_2-t)}\mathbf{B}\right\|_{\infty,t\in[t_1,t_2]} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ t_5 \left\|\mathbf{C}e^{\mathbf{A}(t_5-t)}\mathbf{B}\right\|_{\infty,t\in[t_0,t_1]} & t_5 \left\|\mathbf{C}e^{\mathbf{A}(t_2-t)}\mathbf{B}\right\|_{\infty,t\in[t_0,t_2]} & \cdots & t_5 \left\|\mathbf{C}e^{\mathbf{A}(t_5-t)}\mathbf{B}\right\|_{\infty,t\in[t_0,t_5]} \end{bmatrix} = \begin{bmatrix} 1.904 & 0 & \cdots & 0 \\ 3.397 & 3.380 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 7.481 & 7.648 & \cdots & 9.521 \end{bmatrix} \qquad (24)$$

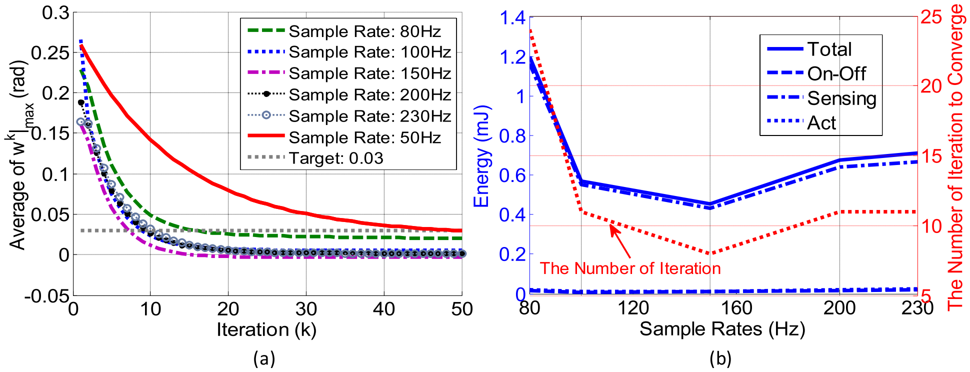

Fig. 2. (a) Estimated convergence rate with respect to sample rate, the changeable control setting for the heuristic search adaptive controllers. (b) Energy dissipation to reach the target error with respect to sample rates (Act: actuator energy. ON-OFF: switching drive circuit energy. Sensing: sensor circuit energy. Total: total energy for actuation, drive circuit, and sensing).

Adaptive controller performance has also been verified experimentally, and experimental convergence rates are also available. Refer in [4] for detail system description of the experimental system. A sample result is shown in black lines of Fig. 1(a). This result demonstrates a successful estimation of convergence rate with the maximum bounded error. In this results, the estimated error are converged to zero from 20 iterations with $N = 5$ measurements and $Q = 2$ transitions per measurement.

As can be seen in black lines of Fig. 1(a), the experimental system converges faster than the estimated convergence rate, but to various error levels with the same order of iterations. The faster convergence rate is likely a consequence of having just a single, essentially random, test case to evaluate experimentally, though it may reflect nonlinearities incorporated into evaluation of convergence or nonconvergence from [4], but not the convergence rate estimator.

3) Estimation of Energy Dissipation: Once predictions of controller convergence behavior can be made for various selections of adaptation gains, numbers of switching instances, and sampling times, it becomes possible to identify strategies for reducing total control system energy usage. For instance, Fig. 2(a) shows the estimated convergence rate for the ON-OFF controller applied to the test system with various sampling frequencies, and two switching transitions between each measurement. As can be seen for a given final error target, sampling at higher frequencies tends to result in adaptation to the desired error level over a smaller number of iterations. However, increasing sampling rate increases power consumption, and there exists a sample rate such that convergence speed no longer significantly improves. It is also noted that the convergence bound becomes very loose should sampling take place below the Nyquist rate for a given system.

With the results in Fig. 2(a) and for a given sensing architecture, energy dissipation to reach the target error versus sample rate is obtained, as shown in Fig. 2(b). The total and sensing energy use decrease from lower sample rates to 150-Hz sample rate as the number of iterations for convergence decreases. Then, the total energy increases as increased power consumption of faster sampling fails to significantly improve time to convergence. Only slightly more driving energy is required throughout the sampling rate increase. Thus, convergence rate knowledge can be used to estimate optimal sampling rates. For the example system, for instance, a 150-Hz sample rate may be suggested as the proper sampling rate. In addition, as mentioned the *prior* section, a slower sample rate (50 Hz) than Nyquist rate is still feasible but the convergence bound is much looser, as shown in Fig. 2(a), due in large part to a very large of $\mathbf{F}$ matrix. The derivation of convergence rate for this case is still valid but usefulness is more limited.

B. Case II: Adaptive ON-OFF *Controller Based on Stochastic Approximation*

1) Test System Description: A second approach to ON-OFF adaptive control that has the potential to further reduce sensor energy consumption is the use of gradient estimation. The controller has been described in [3], with Simultaneous Perturbation Stochastic Approximation (SPSA) used to minimize an object function with just a single-sensor measurement in each iteration. Thus, in this case, the sampling rate is fixed at $N = 1$, but the number of switches Q can vary. The adaptation rules for this controller can be rewritten as

$$\Psi_{q,1}^k = -\frac{a^k}{c^k \Delta_{q,1}^k} J(\boldsymbol{\tau}^k), \quad q = 1, \ldots, 4 \tag{25}$$

where a^k and c^k are sequential gain coefficients of the form $a^k = a_0/(k + K)^\alpha$ and $c^k = c_0/k^\gamma$ and $\{\boldsymbol{\Delta}^k \in \{-1, +1\} | \boldsymbol{\Delta}^k = [\, \Delta_{1,1}^k \cdots \Delta_{4,1}^k]^T \}$ is a vector of four-independent binomial zero-mean random variables. Variables a_0, c_0, K, α, and γ are constants from the SPSA algorithm.

Under specific properties of the dynamic system to be controlled, (10) can be true using SPSA algorithms [25] so that the expected monotonic condition can be satisfied to apply the proposed convergence rate estimation. These conditions are properties of measurement noise, smoothness and boundedness of cost function, statistical properties of perturbed variables, and asymptotic normality. The detailed conditions of properties and theoretical proof of (10) are described in [23] and [24]. In addition, in prior work [3] it was described how these conditions can be satisfied for ON-OFF control of a microactuator, with appropriate noise conditions a reasonable description of the microrobotic leg test case.

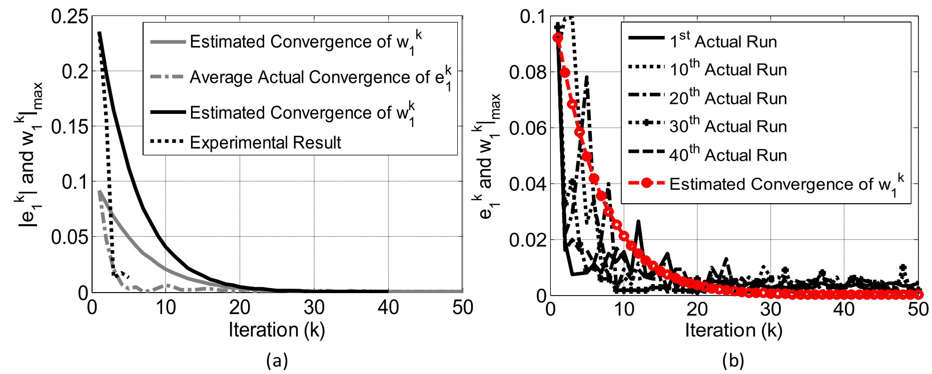

Fig. 3. (a) Average versus estimated convergence rate for the adaptive controller with SPSA (red lines) and experimental versus estimated convergence rate for the adaptive controller (blue lines). (b) Several single runs of actual convergence rate for the adaptive controller with SPSA.

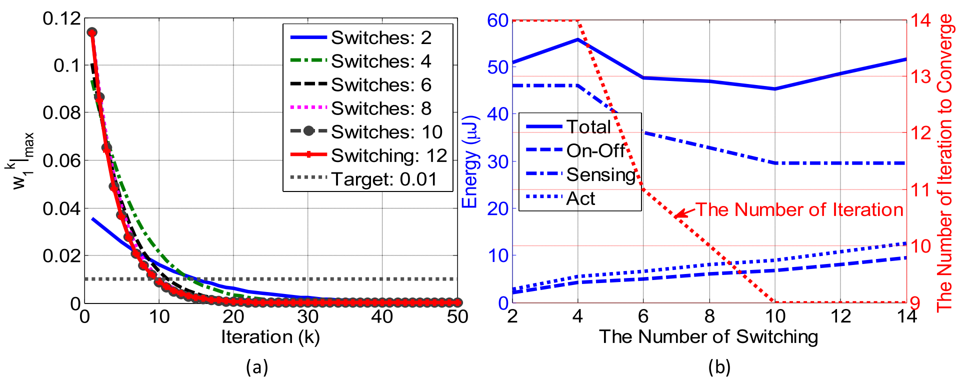

Fig. 4. (a) Estimated convergence rate with respect to the number of switching instance, the changeable control setting for the adaptive controller with SPSA. (b) Energy dissipation to reach the target error with respect to the number of switches per iteration (Act: actuator energy. ON-OFF: switching drive circuit energy. Sensing: sensor circuit energy. Total: total energy for actuation, drive circuit, and sensing).

2) Convergence Rate Analysis: The stochastic controller was applied to a microrobot leg joint with nominal dynamics:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -58.13 & -3.06 \times 10^6 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 13.5 \times 10^5 \\ 0 \end{bmatrix} u(t), z(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (26)$$

The $E[\Gamma]$ and its value for the simulated example when $Q = 1$ and $k = 1$ are formulated as follows:

$$E\left[\Gamma^k\right] = Q\frac{a^k}{c^k}E\left[\frac{1}{\Delta_{q,1}}\right] = 0.1318$$
$$\mathbf{F} = t_1 \left\| \mathbf{C}e^{\mathbf{A}(t_1-t)}\mathbf{B} \right\|_{\infty, t\in[t_0,t_1]} = 1.1518. \quad (27)$$

In simulation, the expected error bound (15) for the test case is shown in gray lines of Fig. 3(a). In this result, values used for a_0, c_0, α, γ, and K are $1.3e^{-5}$, $5e^{-5}$, 0.602, 0.101, and 2000, respectively. Target displacement was 0.3 radians. As can be seen, as the controller is iterated average errors of output from 40 test cases decrease and actual error is bounded by the estimated error. Because of the stochastic nature of the controller, on a case by case basis violations of the error bound may occur. This is shown in Fig. 3(b), where several individual simulation runs are displayed. Even though violations exist, the estimator, however, still predicts quiet well the general convergence behavior, and thus is still effective for evaluating trends in convergence rate by the proposed SPSA approach. Again, performance of the proposed method to estimate convergence rate was also verified experimentally for the adaptive controller with SPSA. Black lines in Fig. 3(a) show a sample experimental and estimated convergence rate. The estimated bound of convergence rate shows a similar trend to some of the faster convergence results from the simulation results, within the estimated convergence bound. Only the single experimental test shown is currently available because controller implementation to test effectiveness of the SPSA approach used a predetermined sequence of random Δ. This was done to mimic limitations of on-board processing capabilities in autonomous microrobots, but as a result, the earlier simulation study is regarded as a more accurate indication of convergence rate estimation effectiveness.

3) Estimation of Energy Dissipation: In this control architecture, the primary tradeoff between convergence performance and energy dissipation arises from the number of switching instance to adapt during each iteration, since only a single-sensor measurement is taken in all cases. Fig. 4(a) shows the estimated convergence rate for the ON-OFF controller applied to the test system with various switching transitions and one measurement. As can be seen, more switching instances produce convergence to reach a desired error level over less iteration. However, improvement in convergence becomes insignificant beyond a certain number of iterations (~8 in this case).

To identify a desired number of switching instances to use with respect to energy usage, sensing and driving energy must be compared. Sensor energy increases with the number of iterations needed, and thus decreases until ~10 switching instances are used, and convergence improvement is minimal. Energy used to drive the actuators increases proportionally with switching instances. Thus, since the total energy change is dominated by the driving energy, an optimal choice of switching instances can be found as shown in Fig. 4(b).

VI. Conclusion

An approach to predicting convergence rate of certain types of iterative adaptive ON-OFF controller is presented. Once predictions of controller convergence behavior for various selections of adaptation gains, numbers of switching instances, and sampling times can be made, it becomes possible to identify strategies for reducing total control system energy usage when identifying optimal input sequences for repeated actuator motions. For example, in test cases with two different ON-OFF controllers, it is found that taking larger numbers of sensor samples and allowing the controller to switch more frequently reduce the number of iterations for convergence. However, since each of these changes increase energy consumption within individual iterations, optimal sampling rates or switching quantities, respectively, can be estimated. In addition, there exist saturated values of sampling rates and switching instances beyond which convergence rate is not further improved.

There are various limitations for convergence rate analysis. The most restrictive is that a true bound on error is only available when the adaptation law can be designed so that all switching times are converging. While this is possible in the regions of convergence of the two controllers presented, due to the methods in which there original convergence was proven, it can be difficult to confirm for other controllers of cases of many switching instants. On the implementation side of the heuristic adaptive controller, use of fixed adaptation gains causes some oscillation in error to be frequently observed at small error levels, as the assumption in (10) is often no longer maintained in practice. Meanwhile, when a nondeterministic controller is analyzed, as in the second case, the error bound may be violated within individual iteration runs, although on average the error estimates provide good predictions of controller behavior.

Nonetheless, the convergence analysis provided here should provide a useful tool for energy management when controlling microactuators operating in uncertain but only slowly varying environments. Over a finite number of iterations, suitable switching instances of an ON-OFF controller may be obtained, and then used for many additional repeated motions before restarting sensor measurements and input adaptation. The authors view this approach as especially desirable for autonomous microrobots making hundreds or thousands of stepping motions with very limited power availability. Furthermore, it is observed by the authors that both convergence and accurate prediction of convergence rate are often seen outside the ranges of guaranteed convergence obtained using the methods described, even though successful adaptation over these larger ranges of initial errors in switch timing cannot currently be predicted analytically.

In the future, the estimated convergence rates for various controller parameters may be integrated with analyzes of other candidate sensor and driving circuits' power usage, to identify improve robot control circuitry configurations, and corresponding controller settings minimizing total energy use. In addition, an efficient method for jointly optimizing both adaptation gains (currently held constant) as well as sampling and driving schemes would provide greater flexibility in controller design. Finally, it would be useful to find rapid methods for computing the gain matrices used in the analysis, to allow simpler practical use.

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